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Well-Covered Claw-Free Graphs

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We prove the existence of a polynomial time algorithm to tell whether a graph, with no induced subgraph isomorphic to $K_{1,3}$, is well covered. A graph is well-covered if all its maximal independent sets are of the same cardinality. The problem

1. Background and Motivation

A graph G is said to be *well-covered* if every maximal independent set of vertices in G is also maximum. This notion was introduced by Plummer in [7]. Several results on the subject have been published since then and a thorough review can be found in [6].

Clearly, a graph is well-covered if and only if the greedy algorithm for constructing a maximal independent set always results in a maximum independent set. In that sense, the family of well-covered graphs forms the set of *greedy instances* of the maximum independent set problem. Greedy instances of other combinatorial problems can be similarly defined. A recent paper of Caro, Sebő and Tarsi [1] investigates the complexity of recognizing greedy instances of several combinatorial problems, one of which is the construction of maximum independent sets. The recognition of well-covered graphs was recently proven to be Co-*NPC*, independently by Chvátal and Slater [2], Sankaranarayana and Stewart [8] and by Caro, Sebő and Tarsi [1]. The last reference presents a stronger version, where the input graph is restricted to have no $K_{1,4}$ induced subgraph:

THEOREM 1.1. *The following decision problem is Co-NP-complete: Input: A graph G with no induced subgraph isomorphic to $K_{1,4}$. Question: Is G well-covered?*

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The significance of the forbidden induced subgraph lies in comparing such results to the analogous problems on line-graphs. A matching in a graph can be viewed as an independent set of vertices in its line-graph. Hence, any problem on maximum independent sets, when restricted to line-graphs, provides an analogous maximum matching problem. Line-graphs are known to be characterized by a list of forbidden induced subgraphs, one of which is the *claw*-$K_{1,3}$ (see e.g. [4]). Line-graphs then, form a subset of the larger family of *claw-free graphs* (graphs with no $K_{1,3}$ induced subgraph). Whenever a combinatorial problem, involving independent vertex sets, is faced, which is "hard" (say NP-hard) for general graphs, and "easy" (say polynomial) for line-graphs (matching), a natural step is to study its complexity, when restricted to claw-free graphs on one hand, and to $K_{1,4}$-free graphs on the other.

An example of the above is the maximum independent set problem which is NPC in general, versus the polynomially solvable maximum matching problem. Here the problem remains NPC when restricted to $K_{1,4}$-free graphs and it is polynomial for claw-free graphs:

THEOREM 1.2. *The maximum independent set problem* (*telling whether a given graph admits an independent set of a given size*) *remains NPC, when restricted to $K_{1,4}$-free graphs.*

We consider the above as "folklore". It is mentioned in [5] and can easily be derived from 3-dimensional matching.

Polynomial time algorithms to construct a maximum independent set in a claw-free graph were developed by Minty [5] and by Sbihi [9]. Sbihi's algorithm directly tackles the input graph, while Minty first reduces the problem to one on a line-graph and then applies any maximum matching algorithm. A similar, yet simpler scheme was then presented by Lovàsz and Plummer in the last section of their book [4]. Our work strongly relies on that scheme. Minty's more complicated algorithm has the advantage of solving the more general *weighted* version of the problem, which we also use as a tool in the sequel:

THEOREM 1.3. [5]. *Let $w: V \to R^+$ be a positive weight function on the vertex set of a claw-free graph $G=(V,E)$. The weight of a set $V' \subseteq V$ is $w(V') = \sum_{x \in v'} w(x)$. A maximum weight independent set of vertices can be constructed in polynomial time.*

The recognition of well-covered graphs presents a similar pattern: As stated in Theorem 1.1, the general problem, as well as its restriction to $K_{1,4}$-free graphs is Co-NPC. The line-graph of a graph G is well-covered if and only if all maximal matchings of G are maximum. Lesk, Plummer and Pulleyblank [3] developed a polynomial time algorithm to recognize such graphs, which they referred to as *equimatchable.*

THEOREM 1.4. [3]. *There exists a polynomial time algorithm which decides whether a given input graph is equimatchable.*

The main result of this paper is the following stronger statement:

THEOREM 1.5. *There exists a polynomial time algorithm which decides whether an input claw-free graph is well-covered.*

2. PROOF OF THEOREM 1.5

2.1 *General Scheme*

In Section 12.4 of their book [4] Lovàsz and Plummer present an algorithm to construct a maximum independent set of a claw-free graph G. Their algorithm is based on the following scheme: G is transformed into a new graph $\bar{G}$ which is the line-graph of some graph H. The graph H is then constructed and a maximum independent set of $\bar{G}$ is found by any maximum matching algorithm on H. The way $\bar{G}$ is obtained from G allows keeping track of how the size of a maximum independent set is changed (in practice always decreased. That way, once a maximum independent set of $\bar{G}$ is found, it can be efficiently extended into a maximum independent set of G.

If there is a way to guaranty that, in addition to the above, also $\bar{G}$ is well-covered if and only if G is well-covered, then this property can be decided by the equimatchable algorithm of Lesk, Plummer and Pulleyblank, (Theorem 1.4), applied to the graph H.

More specifically, $\bar{G}$ is the last term in a sequence of claw-free graphs $\{G_0 = G, G_1, \ldots, \bar{G}\}$ each of which is obtained from its predecessor, until a line-graph $\bar{G}$ is reached. Checking the details of the Lovàsz and Plummer construction, one can observe that, indeed, if G_i is well-covered then this property also holds for G_{i+1}. The converse, however, is false (the 5-wheel, for example, is transformed into an empty graph). To overcome this, we developed a polynomial procedure, which checks the graph G_i and returns either one of the following outputs:

1. "G_i is not well-covered", or
2. "G_i is well-covered, if and only if G_{i+1} well-covered."

This procedure is executed before each step of the construction. If output 1. is ever obtained then the algorithm halts, announcing that G is not well-covered. Otherwise, the line-graph $\bar{G}$ is finally tested as mentioned above.

Let $G=(V,E)$ be a graph, $a\in V$ a vertex of G and $A\subseteq V$ any set of vertices. Here are some definitions that we use in the following:

The neighborhood $N(a)$ of a is the set of all vertices adjacent to a
$\bar{N}(a)=N(a)\cup\{a\}$
$\bar{N}(A)=\bigcup_{a\in A}\bar{N}(a)$
$N(A)=\bar{N}(A)\backslash A$
$N_2(a)=N(\bar{N}(a))$
$\bar{N}_2(a)=\bar{N}(\bar{N}(a))$
$\alpha(A)$ is the cardinality of a maximum independent subset of A
a set of vertices A dominates another set B if $B\subseteq\bar{N}(A)$.

2.2. *Eliminating Irregular Vertices*

A vertex a of a claw-free graph is *regular* if $N(a)$ is the union of two cliques. Otherwise a is *irregular*. Every vertex of a line-graph is clearly regular. The reduction of Lovàsz and Plummer consists of two main phases. The first is aimed toward the elimination of irregular vertices as follows: Let G be a claw-free graph and a, an irregular vertex of G. Let Y be the set of those vertices $y\in N_2(a)$ for which $N(a)\backslash N(y)$ induces a clique in G. Let G' be the graph obtained from G by deleting $\{a\}\cup N(a)\cup Y$ and by joining every two (as yet non-adjacent) vertices of $N_2(a)\backslash Y$.

The following properties of this construction are all proven in Section 12.4 of [4]:

PROPOSITION 2.1. *G' is claw-free*

PROPOSITION 2.2. $\alpha(N_2(a))\leqslant 2$

PROPOSITION 2.3. $\alpha(\bar{N}_2(a))\leqslant 3$

PROPOSITION 2.4. $\alpha(\{a\}\cup N(a)\cup Y)\leqslant 2$

Let us state one more simple observation, which is not explicitly mentioned in [4].

PROPOSITION 2.5. *None of the vertices of $\{a\}\cup N(a)\cup Y$ (deleted when G is reduced to G') is adjacent to a vertex in $V\backslash\bar{N}_2(a)$.*

Indeed, since the vertices of $N(a)$, which are non-adjacent to a fixed $y\in Y$, form a clique and, by irregularity of a, $N(a)$ is not the union of two cliques, then there exist two non-adjacent members of $N(a)$, both adjacent to y. An edge between y and a vertex in $V\backslash\bar{N}_2(a)$ would then complete an induced claw. The other deleted vertices, those in $\bar{N}(a)$, are, by definition, adjacent to vertices of $\bar{N}_2(a)$ only.

In what follows G, a, Y, G' are as described above and $N_2(a)\backslash Y$ is denoted by X. For any maximal independent set B of G Let $B \cap \bar{N}_2(a)$ be shortly denoted by $c(B)$.

Our algorithm is based on the following lemmas:

LEMMA 2.1. *Let B be a maximal independent set of G. If one of the following three conditions holds, then G is not well-covered*:

(i) $|c(B)| = 1$

(ii) $|c(B)| = 2$ *and* $B\backslash c(B)$ *does not dominate* X

(iii) $|c(B)| = 3$, $|c(B) \cap X| = 2$ *and for some* $x_1 \in c(B) \cap X$, $B\backslash\{x_1\}$ *does not dominate* $V\backslash\bar{N}_2(a)$

Proof. Suppose that i holds. By irregularity of a there exists two non-adjacent vertices v_1 and v_2 in $N(a)$. By proposition 2.5, none of these two is adjacent to a vertex in $B\backslash c(B)$. Hence, $B' = (B\backslash c(B)) \cup \{v_1, v_2\}$ is an independent set whose cardinality is bigger than $|B|$.

If condition ii holds, then there exists $x \in X$ such that $\{x\}$ is not adjacent to $B\backslash c(B)$. By the definition of Y, for such $x \in X = N_2(a)\backslash Y$, there exist two vertices $v_1, v_2 \in N(a)$ such that $\{x, v_1, v_2\}$ is independent. Hence, $B' = (B\backslash c(B)) \cup \{x, v_1, v_2\}$ is an independent set of cardinality bigger then $|B|$.

Assume now that condition (iii) holds and let $c(B)$ be $\{x, x_1, v\}$, where $\{x, x_1\} \subseteq X$. There exists a vertex $s \in V\backslash\bar{N}_2(a)$, such that s is non-adjacent to any vertex $B\backslash\{x_1\}$. The same argument used in the previous case provides an independent set $B' = (B\backslash\{x_1, v\}) \cup \{v_1, v_2, s\}$, whose cardinality is bigger than $|B|$. In each of the three cases, the maximal independent set B is not maximum and hence G is not well-covered. ∎

LEMMA 2.2. *If there is no maximal independent set of G for which any of the conditions* (i), (ii) *or* (iii) *of Lemma* 2.1 *holds, then G' is well-covered if and only if G is well-covered.*

Proof. We show that for any maximal independent set B of G, there exists a maximal independent set B' of G' of size $|B| = |B| - 2$ and vise versa. Let B be a maximal independent set of G. Since none of the conditions of Lemma 2.1 holds one of the following cases occurs:

1. $|c(B)| = 2$ and X is adjacent to $B\backslash c(B)$. Clearly, in that case, there are no members of X in B, because such vertices would not be adjacent to $B\backslash c(B)$. It turns out that when G' is created, the two members of $c(B)$ are

deleted to form an independent set $B' = B \backslash c(B)$ of cardinality $|B| - 2$. It remains to show that B' is maximal in G': No vertex of X can be added to B', because X is adjacent to $B' = B \backslash c(B)$. By Proposition 2.5, the deletion of vertices did not "free" any vertex of $V \backslash \bar{N}_2(a)$ and hence no such vertex can either be added.

2. $|c(B)| = 3$ and $|B \cap X| = 1$. Here, again, two vertices are deleted from B when G' is formed and an independent set B' of G' is obtained. No vertex of X can be added, because there is already one such vertex in B' and X is a clique in G'. The argument used in case 1, regarding $V \backslash \bar{N}_2(a)$, is valid here as well.

3. $|c(B)| = 3$, $B \cap X = \{x_1, x_2\}$ and $V \backslash \bar{N}_2(a)$ is dominated both by $B \backslash \{x_1\}$ and by $B \backslash \{x_2\}$. In that case one vertex of B is deleted when G' is formed and either x_1 or x_2 should be removed too, because these two vertices are adjacent in G'. The obtained independent set B' is maximal, because the removal of either x_1 or x_2 still leaves $V \backslash \bar{N}_2(a)$ dominated by B'.

On the other hand, let B' be a maximal independent set of G'. Since X is a clique in G', there is at most one member of X in B' and thus, two vertices v_1 and v_2 from $N(a)$ can be added to B' to form an independent set B of G. The obtained independent set B includes two vertices of $\bar{N}_2(a) \backslash X$. By Proposition 2.4, B cannot be extended by a third vertex of $\bar{N}_2(a) \backslash X$. If B includes an element of X then it is maximal by Proposition 2.3. If there is no vertex of X in B, it is because B' dominates X and hence it is again maximal. ∎

The following is used to show that the existence of a set B, which satisfies any of the conditions of Lemma 2.1, can be decided in polynomial time.

Lemma 2.3. *Let $G = (V, E)$ be a claw-free graph. Given a set $A \subseteq V$, an independent subset $C \subseteq \bar{N}(A)$ of $\bar{N}(A)$ and a subset T of $\bar{N}(C)$, then the following can be decided in polynomial time: Does there exists a maximal independent set B of G, such that $B \cap \bar{N}(A \cup T) = C$?*

Proof. Let $\bar{N}(A \cup C \cup T)$ be denoted by S. The complement of C to a maximal independent set B, as required, is a maximal independent set I, of $V \backslash S$, which dominates $V \backslash \bar{N}(C)$. It suffices to find such I which dominates $S \backslash \bar{N}(C)$, since it can then be extended to become maximal in $V \backslash S$. Define a non-negative integer weight function w on $V \backslash S$, by $w(y) = |N(y) \cap (S \backslash \bar{N}(C))|$. The set S is the closure ($\bar{N}$) of another set. This implies that every external vertex of S, that is, one which is adjacent to a vertex in $V \backslash S$, is also adjacent to an internal one—a vertex which is not adjacent to any one in $V \backslash S$. Since G is claw-free, no two non-adjacent vertices of $V \backslash S$ are adjacent to the same vertex of S. The weight $w(I)$ of an independent set I

of $V\backslash S$ is then the number of vertices in $(S\backslash\bar{N}(C))$, which are adjacent to I. A set I dominates $S\backslash\bar{N}(C)$, as required, if and only if its weight $w(I)$ equals $|(S\backslash\bar{N}(C))|$. The existence of such set I can be decided bv Minty's algorithm (Theorem 1.3) ∎

LEMMA 2.4. *The existence of a maximal independent set B of G, which satisfies any of the three conditions of Lemma* 2.1, *can be decided in polynomial time.*

Proof. Select some $v\in\bar{N}_2(a)$. Set $A=\bar{N}(a)$, $C=\{v\}$ and $T=\phi$. The existence of a maximal independent set B of G such that $c(B)=\{v\}$ can be decided in polynomial time, by Lemma 2.3. The existence of a set B which satisfies condition i of Lemma 2.1 is decided by checking the above for every $v\in\bar{N}_2(a)$.

Select two non-adjacent vertices v_1 and v_2,$\{v_1, v_2\}\subseteq\bar{N}_2(a)$ and a vertex $x\in X\cap\bar{N}(\{v_1, v_2\})$. Set $C=\{v_1, v_2\}$, $A=\bar{N}(a)$ and $T=\{x\}$. The existence of a set B which satisfies condition (ii) can be decided by applying Lemma 2.3 to all such selections of v_1, v_2 and x.

Condition (iii) is treated in a similar way. Here the selection should be: An independent, subset $C=\{x, x_1, v\}$ of $\bar{N}_2(a)$, where x and x_1 are two non-adjacent members of X, $A=\bar{N}(a)$ and $T=\{t\}$, where t is any vertex of $N(x_1)\cap(V\backslash\bar{N}_2(a))$.

2.3. *Reducible Cliques*

A *reducible clique* in a graph is a maximal clique Q with $\alpha(N(Q))\leqslant 2$. A clique is *irreducible* if it is not reducible. The second phase of the Lovàsz Plummer reduction is based on:

THEOREM 2.1. *Let G be a graph such that every vertex of G is contained in two irreducible cliques which cover all neighbors of the vertex. Then G is a line-graph.*

Theorem 2.1 is proven in [4] (Theorem 12.4.5.). The reduction into a line-graph now proceeds by means of the following construction:

LEMMA 2.5. *Let Q be any reducible clique in the claw-free graph G. Let G' denote the graph obtained from G by deleting the vertices of Q and connecting two as yet non-adjacent vertices u and v of $N(Q)$ by an edge if and only if $Q\subseteq N(u)\cup N(v)$. Then G' is a claw-free graph.*

This is Lemma 12.4.4. of [4], where it is also stated that $\alpha(G')=\alpha(G)-1$. We need a more detailed observation: Let G, Q and G' be as stated in Lemma 2.5. For any maximal independent set B of G let $B\cap\bar{N}(Q)$ be denoted by $c(B)$.

LEMMA 2.6. *If there exists a maximal independent set B of G for which one of the following holds then G is not well-covered:*

(i) $|c(B)| = 1$ *and* $B \backslash c(B)$ *does not dominate* $N(Q)$.

(ii) $c(B) = \{q, u\}$ *where* $q \in Q$, $u \in N(Q)$ *and there exists* $v \in N(Q)$ *non-adjacent to* u *such that* $\{v\}$ *is not dominated by* $B \backslash c(B)$ *and* Q *is not dominated by* $\{u, v\}$.

(iii) $c(B) = \{u, v\} \subseteq N(Q)$, *where* $B \backslash \{u\}$ *does not dominate* $V \backslash \bar{N}(Q)$.

Proof. Let B be a set which satisfies condition (i) of the Lemma. Since Q is a maximal clique, no vertex in $N(Q)$ dominates Q. The single element of B is hence some $q \in Q$. Given that $B \backslash \{q\}$ does not dominate $N(Q)$, there exists $u \in N(Q)$ such that $(B \backslash \{q\}) \cup \{u\}$ is independent. There is also $q' \in Q$ non-adjacent to u and then $B' = (B \backslash \{q\}) \cup \{u, q'\}$ is an independent set of G whose cardinality is bigger than $|B|$.

Assume that condition *ii* holds. None of u and v is adjacent to any vertex in $B \backslash \{q\}$ and $\{u, v\}$ does not dominate Q. Then there exists $q' \in Q$ such that $B' = (B \backslash \{q\}) \cup \{v, q'\}$ independent.

If condition (iii) holds then set $B' = (B \backslash \{u\}) \cup \{s, q\}$, where $s \in V \backslash \bar{N}(Q)$ is not adjacent to $B \backslash \{u\}$ and $q \in Q$ non-adjacent to v. In each one of these three cases, B is not maximum and thus G is not well-covered. ∎

LEMMA 2.7. *If there is no maximal independent set B of G for which one of the conditions of Lemma 2.6 holds then G' is well-covered if and only if G is well-covered.*

Proof. Given any maximal independent set B of G, we will show that there exists a maximal independent set B' of G' with $|B'| = |B| - 1$. There are four distinct cases to check:

1. $|c(B)| = 1$ and $B \backslash c(B)$ dominates $N(Q)$. As already observed, $c(B)$ consists of a vertex of Q and hence it is deleted when Q is removed. The obtained set $B' = B \backslash c(B)$ is independent and since it dominates $N(Q)$ it is maximal in G'

2. $c(B) = \{q, u\}$ where $q \in Q$ and $u \in N(Q)$, but there is no vertex v as required by condition (ii) of Lemma 2.6. Now $B' = B \backslash \{q\}$ is an independent set of G' and it is maximal because if $v \in N(Q)$ is non-adjacent (in G) to any vertex in $B \backslash \{q\}$ then $\{u, v\}$ dominates Q (otherwise (ii) is satisfied), but in that case u and v become adjacent in G' (see the construction in Lemma 2.5).

3. $c(B) = \{u, v\} \subseteq N(Q)$ and $B \backslash \{u\}$ dominates $V \backslash \bar{N}(Q)$. Here again $\{u, v\}$ dominates Q and they are hence adjacent in G'. Clearly $B' = B \backslash \{u\}$ is a maximal independent set of G'.

4. $|c(B)| = 3$. In that case $c(B)$ includes two vertices of $N(Q)$ and one of Q. The first two remain non-adjacent when G' is constructed, while the last one is removed. Since $\alpha(N(Q)) \leqslant 2$ obtained set B' is clearly maximal independent in G'.

Given, on the other hand, a maximal independent set B' of G', a maximal independent set B of G can obtained as follows: If $B' \cap N(Q) = \emptyset$, then select any $q \in Q$ and set $B = B' \cup \{q\}$. If B' contains one vertex of $N(Q)$, then again a vertex of Q can be added since Q is a maximal clique. If there are two vertices $u, v \in N(Q)$ in B' then it is because $\{u, v\}$ does not dominate Q and thus some $q \in Q$ can be added in that case too. The assertion of Lemma 2.7 clearly follows. ∎

LEMMA 2.8. *The existence of a set B which satisfies one of the conditions of Lemma* 2.6 *can be decided in polynomial time.*

Proof. The existence of a maximal independent set B of G with $c(B) = \{q\}$, $q \in Q$ such that some $t \in N(q)$ is not adjacent to $B \backslash \{q\}$ is polynomially decidable, applying Lemma 2.3 with $A = Q$, $C = \{q\}$ and $T = \{t\}$. This should be repeated for every selection of q and t in order to test the existence of a set which satisfies condition (i). Similarly, the appropriate selection when dealing with condition (ii) consists of: two non-adjacent vertices $q \in Q$ and $u \in N(Q)$ and vertex $v \in N(q) \cap N(Q)$ such that u and v are non-adjacent and $\{u, v\}$ does not dominate Q. Lemma 2.3 should be applied with $A = Q$, $C = \{q, u\}$ and $T = \{v\}$. For condition (iii) select $C = \{u, v\}$, an independent subset of $N(Q)$ which dominates Q. Set $A = Q$ and $T = \{s\}$ for some $s \in N(u) \cap (V \backslash \bar{N}(Q))$. ∎

2.4. *The Algorithm*

Let us conclude the proof with a brief sketch of the algorithm as a whole:

Let $G = (V, E)$ be the input graph. Check the vertices, one after another for regularity. Once an irregular vertex a is encountered, apply Lemma 2.4 to check if there exists a set which satisfies any of the conditions of Lemma 2.1, in which case G is not well-covered as the algorithm halts. If there is no such set, then use Lemma 2.2 to set $G := G'$, where G' is the graph obtained by the construction defined in the first paragraph of Section 2.2. If vertex v is regular and hence included in two maximal cliques whose union is $\bar{N}(v)$, then check if any of these two cliques is reducible. Once a reducible clique is found that way, search for a set B which satisfies any of the conditions of Lemma 2.6. This can be done in polynomial time by Lemma 2.8. If there exists such a set then G is not well-covered and the algorithm halts. Otherwise, set $G := G'$ where G' is the graph obtained by the construction described in Lemma 2.5. This is a legal step in that case, by Lemma 2.7. Repeat this until $\bar{N}(v)$ is the union of two irreducible cliques, for every vertex v of G. By Theorem 2.1, G is now a line-graph of a graph H. The graph H can be easily constructed since its vertices are defined by the irreducible cliques of G, just mentioned. G is well-covered if and only if H is equimatchable and this can be decided by the algorithm of Lesk, Plummer and Pulleyblank (Theorem 1.4).

We have made no effort to obtain the most efficient algorithm and thus see no point in an accurate complexity analysis. Every step, however, and the entire algorithm can be easily verified to complete its task within polynomial time bounds.

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